HW #5 SOLUTIONS

Tuesday, May 11, 2010
7:51 AM

10.10 $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{J \cdot d\tau'}{r}$

Since $d\tau' = s \cdot dl$
$\qquad\qquad\;\; \uparrow \text{area} \quad \uparrow \text{length}$

$\vec{J} \cdot d\tau' = \vec{J} \cdot s \cdot dl = I \cdot \vec{dl}$

$$\vec{A} = \frac{\mu_0}{4\pi}\oint \frac{I(t_r)\cdot \vec{dl}}{r} =$$

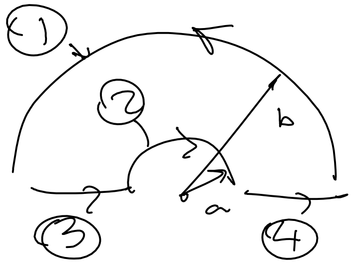
$$= \frac{\mu_0}{4\pi}\oint \frac{k(t - r/c)}{r}\,\vec{dl} =$$

$$= \frac{\mu_0}{4\pi}\left(\oint \frac{kt}{r}\cdot\vec{dl} - \cancel{\oint \frac{k}{c}\cdot\vec{dl}}\right)$$

0

Since $\oint \vec{dl} = 0$
for any closed loop

$$\vec{A} = \frac{\mu_0}{4\pi}kt \oint \frac{\vec{dl}}{r}$$



(1): $\int_{(1)} \frac{\vec{dl}}{b} = -\frac{2b}{b} = -2$

(2): $\int_{(2)} \frac{\vec{dl}}{a} = \frac{+2a}{a} = 2$

(3): $\int_{-b}^{-a} \frac{dr}{|r|} = \ln\frac{b}{a}$

(4) $\int^{b} \frac{dz}{} = \ln\frac{b}{}$

(4) $\int_a^{b} \frac{dz}{|z|} = \ln \frac{b}{a}$

(1) and (2) cancel, (3) & (4) the same:

$$\vec{A} = \frac{\mu_0}{4\pi} kt \left(-2 + 2 + \ln\frac{b}{a} + \ln\frac{b}{a}\right)\hat{x} =$$

$$= \frac{\mu_0 k t}{2\pi} \ln\left(\frac{b}{a}\right) \cdot \hat{x}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0 k}{2\pi} \ln\left(\frac{b}{a}\right) \cdot \hat{x}$$

B is not defined since we can find A only at the center.

Homework #5

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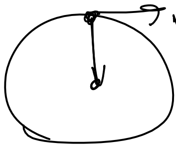
***10.13** Since $\vec{v} \perp \vec{\mathscr{r}}$



$(\vec{\mathscr{r}} = z\cdot\hat{z} + a\cdot\hat{\rho}$

where $\hat{\rho}$ is direction toward center, $\vec{v}\cdot\vec{\mathscr{r}} = z\cdot\underbrace{\vec{v}\cdot\hat{z}}_{=0} + a\,\underbrace{\vec{v}\cdot\vec{\rho}}_{=0})$

$$V(r,t) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{z^2+a^2}}$$

$$\bar{A}(r,t) = \frac{\bar{v}}{c^2}\cdot V(r,t)$$

If position of particle (retarded)

$x = a\cdot\cos\omega t_r$

$y = a\cdot\sin\omega t_r$

where $t_r = t - \frac{\sqrt{z^2+a^2}}{c}$

then velocity (retarded)

$$\bar{v} = \omega a\left(-\sin\omega t_r\cdot\hat{x} + \cos\omega t_r\cdot\hat{y}\right)$$

$$\bar{A} = \frac{q\omega a\left(-\sin\omega t\cdot\hat{x} + \cos\omega t_r\cdot\hat{y}\right)}{4\pi\varepsilon_0 c^2\sqrt{z^2+a^2}}$$

***10.17** $c(t-t_r) = |\mathscr{r}|$

$\frac{\partial}{\partial t}$ both sides:

$$c\,\partial/\partial t\,(t-t_r) = \frac{\partial|\mathscr{r}|}{\partial t} = \frac{\partial\sqrt{\bar{\mathscr{r}}^2}}{\partial t}$$

to get vector $\bar{r}$

$$c\left(1-\frac{\partial t_r}{\partial t}\right) = \frac{1}{2\sqrt{\bar{\mathscr{r}}^2}}\cdot\frac{\partial(\bar{\mathscr{r}}^2)}{\partial t} = \underbrace{\frac{2\bar{\mathscr{r}}}{2|\mathscr{r}|}}_{\hat{\mathscr{r}}}\cdot\frac{\partial\bar{\mathscr{r}}}{\partial t}$$

Since $\bar{\mathscr{r}} = \bar{r} - \bar{\omega}(t)$

$$\frac{\partial\bar{\mathscr{r}}}{\partial t} = \underbrace{\frac{\partial\bar{r}}{\partial t}}_{0} - \frac{\partial\vec{\omega}(t)}{\partial t} = -\frac{\partial\vec{\omega}}{\partial t_R}\cdot\frac{\partial t_R}{\partial t} = -\vec{v}\cdot\frac{\partial t_R}{\partial t}$$

Since $\dfrac{\partial \bar{\omega}}{\partial t_R} = \vec{v}$ (retarded)

$$c\left(1-\frac{\partial t_r}{\partial t}\right) = \hat{r}\cdot\left(-\vec{v}\cdot\frac{\partial t_R}{\partial t}\right)$$

$$\frac{\partial t_r}{\partial t} = \frac{c}{c-\hat{r}\cdot\vec{v}} = \frac{c|r|}{c|r|-\bar{r}\cdot v} = \frac{cr}{\bar{r}\cdot\bar{u}}$$

10.20 $u = c\hat{r} - \vec{v}$

Remember that $\vec{r}\perp\vec{v}$, $\vec{a}\parallel\vec{r}$

$$r\times(u\times a) = (\bar{r}\cdot\bar{a})u - (\bar{r}\cdot\bar{u})\bar{a}$$

$$\bar{r}\cdot\bar{u} = \bar{r}\cdot(c\hat{r}-\bar{v}) = cr$$

$$\bar{r}\cdot\bar{a} = \omega^2 r^2$$

$$r\times(u\times a) = \cancel{c\cdot\omega^2 r^2\cdot\hat{r}} - \underbrace{\omega^2 r^2}_{v^2}\cdot\vec{v} - \cancel{cr\cdot\omega^2\bar{r}}$$

$$E = \frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{(r\cdot u)^3}\left((c^2-v^2)\bar{u} + \bar{r}\times(\bar{u}\times\bar{a})\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{(cr)^3}\cdot\left((c^2-v^2)\cdot c\hat{r} - c^2\bar{v} + \cancel{v^2\bar{v}} - \cancel{v^2\bar{v}}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0 c^2 r^2}\Big[(c^2-\omega^2 r^2)(-\cos\omega t_R\cdot\hat{x} - \sin\omega t_R\cdot\hat{y}) - c\cdot\omega r(-\sin\omega t_R\cdot\hat{x} + \cos\omega t_R\cdot\hat{y})\Big]$$

we used $\hat{r} = -\dfrac{\bar{\omega}}{|\omega|} = -(\cos\omega t_R\cdot\hat{x} + \sin\omega t_R\cdot\hat{y})$

and $\bar{v} = \omega r(-\sin\omega t_R\cdot\hat{x} + \cos\omega t_R\cdot\hat{y})$

$$B = \frac{1}{c}(\hat{r}\times\vec{E})$$

$$\hat{r}\times(c^2-v^2)\cdot c\hat{r} = 0$$

$$\hat{r}\times(-c^2\bar{v}) = -c^2(\hat{r}\times\bar{v}) = c^2\omega r\cdot\hat{z}$$

$$B = \frac{1}{c}\cdot\frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{(cr)^3}\cdot c^2\omega r\cdot\hat{z} =$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{\omega}{c^2 r}\cdot\hat{z}$$

10.21

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda}{\mathscr{r}}\cdot dl = \frac{2}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\lambda_0 a \cdot \sin\frac{\theta}{2}}{a}\cdot d\theta =$$

$$= \frac{2\lambda_0}{4\pi\varepsilon_0}\left(-2\cdot\cos\frac{\theta}{2}\right)\Big|_0^{\pi} = \frac{\lambda_0}{\pi\varepsilon_0}$$

(Here we used the fact that at the center $t_R = t - \frac{a}{c}$ is the same, and integrated from point of zero charge, re-defined as $\theta = 0$ to another such point at $\theta = \pi$ and doubling the result.)

Another way of doing it is to define

$\lambda(\theta, t) = \lambda_0 \left|\sin\left(\frac{\theta - \omega t_R}{2}\right)\right|$ and then

introduce $\varphi = \theta - \omega t_R = \theta - \omega t + \frac{a}{c}$

and $d\varphi = d\theta$

For $\vec{A}$:

$$I(\theta, t) = \lambda \cdot \vec{v} = \lambda_0 \omega \cdot \left|\sin\frac{\overbrace{\theta - \omega t + \frac{a}{c}}^{\varphi}}{2}\right| \cdot \hat{\theta}$$

$$\hat{\theta} = -\sin\theta\cdot\hat{x} + \cos\theta\cdot\hat{y}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\cdot dl}{\mathscr{r}} = -\frac{\mu_0\lambda_0\omega}{4\pi}\int_0^{2\pi} \sin\frac{\varphi}{2}\cdot\left(\sin(\varphi + \omega t_R)\cdot\hat{x} - \cos(\varphi + \omega t_R)\cdot\hat{y}\right)\cdot d\varphi$$

$$\sin A\cdot\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\sin A\cdot\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$\vec{A} = -\frac{\mu_0\lambda_0\omega}{8\pi}\int_0^{2\pi}\left[\cos\left(\frac{\varphi}{2} + \omega t_R\right) - \cos\left(\frac{3}{2}\varphi + \omega t_R\right)\right]\cdot\hat{x}\cdot d\varphi + - \left[\sin\left(\frac{3}{2}\varphi + \omega t_R\right) - \sin\left(\frac{\varphi}{2} + \omega t_R\right)\right]\cdot\hat{y}\cdot d\varphi$$

$$\int_0^{2\pi}\cos\left(\frac{\varphi}{2} + \omega t_R\right)\cdot d\varphi = 2\int_0^{2\pi}\cos\left(\frac{\varphi}{2} + \omega t_R\right)\cdot d\left(\frac{\varphi}{2} + \omega t_R\right) =$$

$$\int_0^{2\pi} \cos\left(\frac{\varphi}{2}+\omega t_R\right)\cdot d\varphi = 2\int_0^{2\pi} \cos\underbrace{\left(\frac{\varphi}{2}+\omega t_R\right)}\cdot d\left(\frac{\varphi}{2}+\omega t_R\right) =$$

$$= 2\cdot \sin\gamma \Big|_{\omega t_R}^{\pi+\omega t_R} = -4\sin\omega t_R$$

Similarly:

$$\int_0^{2\pi} \cos\left(\frac{3}{2}\varphi+\omega t_R\right)\cdot d\varphi = \frac{2}{3}\left(\sin(3\pi+\omega t_R)-\sin\omega t_R\right) =$$

$$= -\frac{4}{3}\cdot\sin\omega t_R$$

$$\int_0^{2\pi} \sin\left(\frac{3}{2}\varphi+\omega t_R\right)\cdot d\varphi = -\frac{2}{3}\left(\cos(3\pi+\omega t_R)-\cos\omega t_R\right) =$$

$$= \frac{4}{3}\cos\omega t_R$$

$$\int_0^{2\pi} \sin\left(\frac{\varphi}{2}+\omega t_R\right) d\varphi = -2\left(\cos(\pi+\omega t_R)-\cos\omega t_R\right) =$$

$$= 4\cdot\cos\omega t_R$$

$$\vec{A} = -\frac{\mu_0\lambda_0\omega}{8\pi}\cdot\left[\left(-4+\frac{4}{3}\right)\sin\omega t_R\cdot\hat{x} - \left(\frac{4}{3}-4\right)\cdot\cos\omega t_R\cdot\hat{y}\right]$$

$$= \frac{\mu_0\lambda_0\omega}{8\pi}\cdot\frac{8}{3}\left(\sin\omega t_R\cdot\hat{x} - \cos\omega t_R\cdot\hat{y}\right) =$$

$$= \frac{\mu_0\lambda_0\omega}{3\pi}\left(\sin\omega\left(t-\frac{a}{c}\right)\cdot\hat{x} - \cos\omega\left(t-\frac{a}{c}\right)\cdot\hat{y}\right)$$

OR $\frac{\mu_0\lambda\omega}{3\pi}\cdot(-\hat{\varphi}')$ where $\varphi = \omega t_R = \omega\left(t-\frac{a}{c}\right)$